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INFRARED EFFECTS AND THE ASYMPTOTICS OF PERTURBATION THEORY IN WEAK DECAYS OF HEAVY PARTICLES

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ABSTRACT

I discuss the interplay of infrared sensitivity in large order perturbative expansions with the presence of explicit nonperturbative corrections in the context of heavy quark expansions. The main focus is on inclusive decays and the status of the kinetic energy of the heavy quark. This talk summarizes work done with Braun and Zakharov.

1. Introduction

The study of infrared (IR) divergences in perturbation theory (PT) is crucial to any rigorous approach to hard processes in QCD. The appearance of an explicitly divergent *coefficient* (say, $\ln \lambda^2$, if a finite gluon mass is used as regulator to leading order) indicates that the process can not be calculated perturbatively. However, for a wide class of phenomena, the IR divergence is universal and can be factorized into a few nonperturbative functions and process dependent coefficient functions, which are perturbatively calculable. Defying factorization, in *large* orders in PT, the coefficient function is dominated by IR regions of Feynman integrals, typically involving a large number of vacuum polarizations in a gluon line. The corresponding perturbative *series* develops an IR renormalon divergence in large orders, which renders the sum of the series undefined by terms suppressed by a power of the hard scale. Thus the presence of IR renormalons implies the existence of "higher twist" terms and requires the introduction of new nonperturbative parameters, such that the sum of leading and higher twist is unambiguous. [The converse is not true: Power suppressed terms may exist which are not indicated by renormalons in previous orders.] In cases where an operator product expansion (OPE) is available, these parameters are naturally identified with matrix elements of higher dimension operators, but the argument is sufficiently general to comprise situations without OPE.

Practically, all present calculations of large order coefficients are restricted to diagrams with a *single* gluon line, dressed by fermion loops, which is equivalent to

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integrating the gluon with the running coupling at the vertex. IR renormalons are then conveniently discussed in terms of singularities of the Borel transform (BT) $B[\{r_n\}]$ of the series of coefficients $\{r_n\}$ generated in this way. In this approximation, there is a very transparent relation[1] between the singularities of the BT and the low order coefficient $r_0(\lambda)$, regulated with a finite gluon mass:

$$r_0(\lambda) = \frac{1}{2\pi i} \int\limits_{-1/2-i\infty}^{-1/2+i\infty} \mathrm{d}s\, \Gamma(-s)\Gamma(1+s) \left(\frac{\lambda^2}{\mu^2} e^C\right)^s B[\{r_n\}](s) \tag{1}$$

IR renormalon singularities in $B[\{r_n\}]$ are in one-to-one correspondence with *nonanalytic* (in λ^2) terms in the small-regulator expansion of $r_0(\lambda)$. This relation unifies the renormalon phenomenon with the familiar discussion of explicit IR divergences, $\ln \lambda^2$. Beyond the restriction to a single gluon line, a finite gluon mass has to be abandoned as an IR regulator. Within dimensional regularization one might expect a similar correspondence of renormalons with poles in different from four dimensions, though a precise relation analogous to Eq. (1) has not yet been established.

In the following, I give a brief summary of results that have been obtained applying these general ideas to weak decays of heavy hadrons. In this case the hard scale is provided by the mass of the heavy quark.

2. HQET and the pole mass

Consider the heavy mass expansion of, say, the B meson mass:

$$m_B = m_b^{\text{pole}} + \bar{\Lambda} + O\left(1/m_b^{\text{pole}}\right) \tag{2}$$

The first term is given by the *pole* mass of the heavy quark (HQ), which is therefore the natural expansion parameter for heavy quark effective theory (HQET). The pole mass is IR finite, but turns out to be linearly sensitive to IR momenta. Consequently, the series that relates m_b^{pole} to $m_b^{\overline{\text{MS}}}$ (which in principle can be measured to arbitrary accuracy) has an IR renormalon such that the pole mass is not defined to an accuracy better than Λ_{QCD} within PT.[2,3] This is not unexpected, since $\bar{\Lambda}$ is expected to be of this order. However, the divergence in the leading term m_b^{pole} implies that $\bar{\Lambda}$ is not defined by Eq. (2) by terms of the same order of magnitude, Λ_{QCD}, and only the sum is physical (up to higher orders in $1/m_b^{\text{pole}}$). It follows that HQET does not provide a *unique* nonperturbative definition of the concept of the pole mass.

3. Exclusive decays

The parameter $\bar{\Lambda}$ appears (together with new form factors) in the leading finite mass corrections to the HQ limit of the matrix elements relevant to exclusive decays. To display the implications of an ambiguous nature of $\bar{\Lambda}$, the decay $\Lambda_b \to \Lambda_c l \bar{\nu}$ is simplest, since finite mass corrections involve $\bar{\Lambda}$ alone and no new form factors.[4] The renormalon ambiguity of $\bar{\Lambda}$ is fixed already by m_b^{pole}. Since the physical matrix element must be unambiguous, a consistency relation emerges: The series of radiative corrections to the HQ limit must have a renormalon that matches the ambiguity of $\bar{\Lambda}$. This renormalon

arises, because the leading term in the effective Lagrangian reproduces correctly only the leading IR contribution, $\ln \lambda^2$, of the full theory matrix element. Thus, the matching coefficient is IR finite, but contains $\sqrt{\lambda^2}$. The coefficients have been calculated[5] and satisfy the consistency relations.

Thus, when $\bar{\Lambda}$ is eliminated in the relation of physical quantities, no ambiguities remain. On the other hand, if one attempts to *calculate* $\bar{\Lambda}$, e.g. from QCD sum rules,[6] the ambiguity in the definition Eq. (2) can not be avoided and is indeed consistently reflected in the sum rules.[2] However, both sum rules and phenomenology point towards a large value, $\bar{\Lambda} \approx 500$ MeV, and one may argue that this value is larger than the renormalon ambiguity. This is not unreasonable, because $\bar{\Lambda}$ contains the spectator contribution to the meson mass in the first place, which is not related to renormalons at all. From this point of view the ambiguity inferred from renormalons can serve as an intrinsic "error bar" on $\bar{\Lambda}$. Whether this picture is stable numerically, can only be decided by comparison with phenomenology.

4. Inclusive decays

Significant progress has been made over the past years, applying heavy quark expansions to inclusive heavy flavour decays, e.g., the semileptonic decay $B \to X_q l \bar{\nu}$. Within the OPE and HQET, one finds that Λ_{QCD}/m_b corrections are absent[7,8] and second order corrections can be parameterized by the kinetic and chromomagnetic energy of the heavy quark inside the meson, μ_K and μ_G. The leading term in the decay width coincides with that of a free quark to all orders in PT and it appears natural to use the pole mass in

$$\Gamma_B = \frac{G_F^2 \left(m_b^{\rm pole}\right)^5}{192\pi^3} (1 + \text{radiative corr.} + \text{nonpert. corr.})\,. \tag{3}$$

However, from Sect. 2 above, one concludes that the pole mass has a large distance ambiguity of order $\Lambda_{\rm QCD}$ in apparent conflict with the absence of a nonperturbative parameter that could absorb a $\Lambda_{\rm QCD}/m_b$ correction in Eq. (3). Thus one might ask whether the short distance expansion (OPE) provides the correct *normalization* of Γ_B, given a situation, where a coloured particle (the b-quark) lives long in the initial state. To clarify this question, one has to identify the leading renormalons (or IR sensitive contributions) in the large order radiative corrections to the tree decay width Γ_0 in Eq. (3). Using Eq. (1), one may take a finite gluon mass to tag renormalons and finds[1]

$$\frac{1}{\Gamma_0}\frac{{\rm d}\Gamma}{{\rm d}x} = \Theta(x)\Theta(1-x)\left[6x^2-4x^3+\frac{\alpha}{3\pi}\left\{F(x)+\frac{\lambda}{m_b}\left(24x^2-8x^3\right)\right\}\right]+\frac{4\alpha}{3\pi}\frac{\lambda}{m_b}\,\delta(1-x) \tag{4}$$

for the lepton spectrum, where $x = (2E_l)/m_b^{\rm pole}$, E_l the lepton energy and $F(x)$ is the one-loop radiative correction for $\lambda = 0$. The δ-function appears, since close to the endpoint the gluon mass is no longer small compared to the invariant mass of the hadronic final state. One may now use $m_b^{\rm pole} = m_b^{\overline{\rm MS}} - (2\lambda\alpha)/3$ to eliminate the pole mass in favour of a mass parameter that is not linearly sensitive to large distances (such as $m_b^{\overline{\rm MS}}$) in Eq. (4). Then all linear in λ terms disappear from the spectrum and

consequently the total width, implying cancellation of the renormalon in the pole mass that could have indicated a $\Lambda_{\rm QCD}/m_b$ correction with a renormalon in the radiative corrections to the free quark decay.

Extending Eq. (4), the subsequent nonanalytic terms $\lambda^2 \ln \lambda^2$ have also been found to vanish.[1] In view of Eq. (1), it follows that the summation of large order radiative corrections to the free quark decay is free from ambiguities up to third order in $1/m_b$ or higher. In contrast to $\bar{\Lambda}$ the kinetic and chromomagnetic energy that appear in second order are free from renormalon ambiguities. We may argue that this is true beyond the approximation to which explicit calculations have been performed. The chromomagnetic energy is related to the mass splitting of vector and pseudoscalar mesons and trivially free from ambiguities. For the kinetic energy μ_K this becomes transparent, if one visualizes the heavy mass expansion of the width as a two step process.[8] First one expands the product of two weak Lagrangians at short distances. The short distance mass is the natural mass parameter and the kinetic energy operator does not appear. Second one uses HQET (with the pole mass) to expand the matrix elements. μ_K arises from the expansion of $\langle B|\bar{b}b|B\rangle/(2m_B)$, whose leading term is fixed to unity by current conservation. Thus, neither step can introduce a series of radiative corrections with a divergence corresponding to an ambiguity in μ_K. Another way to see this is to observe that the kinetic energy arises from boosting the free quark decay to an average frame of the heavy quark inside the meson. The kinetic energy is therefore protected by Lorentz symmetry or, in the language of HQET, reparameterization symmetry.[9]

The unambiguous nature of μ_K is important in two respects: First, it is a prerequisite to uphold inequalities such as $\mu_K^2 > \mu_G^2$, that have been derived in various ways[10] in the presence of renormalization. Second, the calculation of subleading nonperturbative parameters in HQET on the lattice has faced serious obstacles in the form of power divergences.[11] While these in general are closely related to the emergence of renormalons in the continuum,[2] for the particular case of μ_K, mixing with lower dimension operators appears to be a genuine lattice problem: Reparameterization symmetry which protects μ_K in the continuum, is broken on the lattice.

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